

Improved Analysis of Heat Pulse Signals for Soil Water Flux Determination

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Soil water flux (J) can be estimated from the velocity (V) of a pulse of heat introduced into the soil. Here we consider a method in which V is measured with a three-probe sensor. The center probe heats the soil, and the outer probes measure temperature increases downstream (T_d) and upstream (T_u) from the heater. An equation was recently proposed for approximating J from the ratio T_d/T_u . In this note we show that the accuracy of this equation can be improved by adding a term to correct for the time dependence of T_d/T_u . This term is simple to evaluate and requires no additional measurements. Example calculations (three cases) are used to evaluate improvement in accuracy. When T_d/T_u is measured at a time of 45 s, relative errors in flux estimates are reduced from 10.5, 2.6, and -10.5% to 0.23, 0.06, and -0.23% , respectively, by using the correction term.

REN ET AL. (2000) PRESENTED a heat-pulse method for measuring soil water flux. The method uses a sensor with three parallel, equidistant, cylindrical probes lying in a common plane. The center probe is used to introduce a pulse of heat, and the outer probes are used to monitor changes in temperature upstream and downstream from the heater. The maximum difference between the temperature changes recorded at the upstream and downstream locations is used to quantify the velocity of the heat pulse, which is used to estimate the soil water flux.

Motivated by a desire to simplify the procedure for estimating soil water flux with the three-probe sensor of Ren et al. (2000), Wang et al. (2002) proposed using the ratio of temperature increases at the upstream and downstream locations to estimate the velocity of the heat pulse. The advantage of this approach is that it results in an asymptotic solution of simple algebraic form that can be used to approximate the relationship between the temperature increase ratio and the soil water flux. Although the asymptotic solution has been used with some degree of success in experimental work (Gao et al., 2006; Mori et al., 2003; Ochsner et al., 2005), the fact that it yields only an approximate relationship means that

its use introduces systematic error in flux estimates. As we demonstrate in this note, the magnitude of the error in flux estimates is sufficiently large to warrant concern.

In this note, we introduce an expression that more accurately approximates the relationship between the temperature increase ratio and soil water flux for the three-probe sensor of Ren et al. (2000). Our objective in deriving this expression is to achieve improved accuracy without sacrificing the advantage of simple algebraic form.

THEORY

We begin with the heat equation

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - V \frac{\partial T}{\partial x} \quad [1]$$

in which T is temperature, t is time, κ is the soil thermal diffusivity, V is the heat pulse velocity, and x and y are space coordinates. This is the governing equation for coupled conduction and convection of heat in homogeneous, isotropic soil through which water moves at a constant rate in the x direction. The heat pulse velocity V is related to the soil water flux J by the expression

$$V = \frac{C_w J}{C} \quad [2]$$

where C_w is the volumetric heat capacity of water, and C is the volumetric heat capacity of the soil.

Exact Model

Ren et al. (2000) derived an analytical solution of Eq. [1] to model temperature changes caused by the release of heat from their three-probe sensor. This solution assumes the heater probe to be a line source of infinite length from which heat is released into an unbounded medium with uniform initial temperature. Heat is released at the constant rate q' (W m^{-1}) during the time interval $0 < t \leq t_0$, where t_0 is the heating duration. That is, heating begins at time $t = 0$ and continues until time $t = t_0$. The general solution given by Ren et al. (2000) (see their Eq. [7]) assumes that the line source is located at $(x, y) = (0, 0)$ and is oriented normal to the x - y plane. By arbitrarily taking the initial temperature to be $T(x, y, 0) = 0$, the solution $T(x, y, t)$ becomes equivalent to the temperature increase that results from heating of the line source.

The three-probe sensor is oriented relative to the direction of water flow so that the outer probes measure the temperature directly downstream and upstream from the line source. Ren et al. (2000) used their Eq. [7] to obtain expressions for the temperature increase directly downstream from the line source, $T_d(t)$, and directly upstream from the line source, $T_u(t)$. These expressions are Eq. [10] and Eq. [13] of Ren et al. (2000), respectively. Whereas Ren et al. (2000) used the difference $T_d(t) - T_u(t)$ in their work, here we consider the ratio of $T_d(t)$ and $T_u(t)$, which yields the solution

$$\frac{T_d}{T_u}(t) = \frac{\int_0^t s^{-1} \exp \left[-\frac{(x_d - Vs)^2}{4\kappa s} \right] ds}{\int_0^t s^{-1} \exp \left[-\frac{(x_u + Vs)^2}{4\kappa s} \right] ds}; \quad 0 < t \leq t_0$$

$$\frac{T_d}{T_u}(t) = \frac{\int_{t-t_0}^t s^{-1} \exp \left[-\frac{(x_d - Vs)^2}{4\kappa s} \right] ds}{\int_{t-t_0}^t s^{-1} \exp \left[-\frac{(x_u + Vs)^2}{4\kappa s} \right] ds}; \quad t > t_0 \quad [3]$$

where x_d and x_u are the distances from the heater to the downstream and upstream probes, respectively. If the parameters x_d , x_u , t_0 , and κ are known and the temperature increase ratio T_d/T_u is measured at a particular time

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t , a root-finding algorithm can be used to find the value of V that satisfies Eq. [3]. The value of V can then be used in Eq. [2] to obtain an estimate of the soil water flux, provided the parameters C and C_w are also known. Thus, when taken together, Eq. [2] and [3] can be used to estimate the soil water flux from a measurement of T_d/T_u at an arbitrary time. Hereafter, we refer to the combined use of the Eq. [2] and [3] in this manner as the “exact model.” The disadvantage of the model is that it requires numerical evaluation of integrals and the use of a root-finding algorithm.

Model of Wang et al. (2002)

Wang et al. (2002) proposed use of T_d/T_u to estimate soil water flux but used an asymptotic form of Eq. [3] to obtain their analytical expression for T_d/T_u . They showed that for $t > t_0$, Eq. [3] reduces to the simple form

$$\frac{T_d}{T_u} = \exp\left[\frac{V(x_d + x_u)}{2\kappa}\right] \tag{4}$$

as $t \rightarrow \infty$. That is, the ratio T_d/T_u approaches a constant value as $t \rightarrow \infty$. Substituting Eq. [4] into Eq. [2] and rearranging gives

$$J = \frac{2\lambda}{(x_d + x_u)C_w} \ln\left(\frac{T_d}{T_u}\right) \tag{5}$$

where λ is the thermal conductivity. Equation [5] provides an explicit relationship between J and the value of T_d/T_u as $t \rightarrow \infty$. Although measured values of T_d/T_u have been shown to approach a relatively constant value during measurement periods of $50\text{ s} < t < 60\text{ s}$ (Mori et al., 2003) and $40\text{ s} < t < 50\text{ s}$ (Ochsner et al., 2005), these measurements are only approximations of the true asymptotic value of T_d/T_u . Thus, when Eq. [5] is used in this manner, it must be viewed as an approximate relationship between the flux and T_d/T_u .

Improved Model

Although the model of Wang et al. (2002) correctly accounts for the finite duration of the heat input, it fails to account for the time dependence of the temperature increase ratio T_d/T_u . A better approximation of Eq. [3] can be obtained by treating the finite heat input as an instantaneous heat input to facilitate accounting for the time dependence of T_d/T_u . We derive this approximation by using the exact analytical solution for T_d/T_u corresponding to an instantaneous heat input. This solution

$$\frac{T_d}{T_u} = \exp\left[\frac{(x_u + Vt)^2}{4\kappa t} - \frac{(x_d - Vt)^2}{4\kappa t}\right]; \quad t > 0 \tag{6}$$

can be obtained by using Eq. [4] of Marshall (1958) to write expressions for the temperature rise at downstream and upstream positions in response

Table 1. Parameters used to evaluate the accuracy of flux estimates from Eq. [5] and [7]. Shown are specified values of downstream probe spacing (x_d), upstream probe spacing (x_u), and soil water flux (J) for Cases A, B, and C. Parameters held constant for all cases were soil thermal diffusivity ($\kappa = 5.70 \times 10^{-7}\text{ m}^2\text{ s}^{-1}$), soil thermal conductivity ($\lambda = 1.70\text{ W m}^{-1}\text{ K}^{-1}$), soil volumetric heat capacity ($C = 2.98\text{ MJ m}^{-3}\text{ K}^{-1}$), and heating duration ($t_0 = 15\text{ s}$).

Case	x_d	x_u	J	Vt
	mm		cm h^{-1}	m s^{-1}
A	5.90	6.04	4.7	1.83×10^{-5}
B	5.90	6.04	19.1	7.44×10^{-5}
C	6.04	5.90	4.7	1.83×10^{-5}

† Heat pulse velocity (V) for each case was calculated by using Eq. [2] with $C_w = 4.18\text{ MJ m}^{-3}\text{ K}^{-1}$.

to an instantaneous heat input. Although Eq. [6] does not account for the finite duration of the heat input, it does incorporate the time dependence of T_d/T_u . This approximation of Eq. [3] can be further improved by shifting Eq. [6] in time by one half of the heating duration, so that the instantaneous heat input occurs midway between start ($t = 0$) and finish ($t = t_0$) of the finite heat pulse. This type of correction is used routinely in analytical solutions for solute transport (Warrick, 2003, p. 315). Replacing t with $t - (t_0/2)$ in Eq. [6] and substituting the result into Eq. [2] gives

$$J = \frac{2\lambda}{(x_d + x_u)C_w} \ln\left(\frac{T_d}{T_u}\right) + \frac{C(x_d - x_u)}{2C_w[t - (t_0/2)]}; \quad t > t_0/2 \tag{7}$$

We propose Eq. [7] as an alternative to Eq. [5]. The first term on the right-hand side of Eq. [7] is identical to the right-hand side of Eq. [5] and gives the flux associated with the limiting value of T_d/T_u as $t \rightarrow \infty$. The second term on the right-hand side of Eq. [7] corrects for the time dependence of T_d/T_u at intermediate times. When evaluating Eq. [7] in practice, it is important to remember that T_d and T_u represent temperature increases that result from heating of the line source.

MATERIALS AND METHODS

The accuracy of flux estimates obtained using Eq. [5] and [7] was evaluated for three sets of parameters. The parameters for Cases A and B (Table 1) correspond to results of Ochsner et al. (2005) for Clarion sandy loam (fine-loamy, mixed, superactive, mesic Typic Hapludoll) with fluxes specified as $J = 4.7\text{ cm h}^{-1}$ and $J = 19.1\text{ cm h}^{-1}$. The parameters for Case C are identical to those for Case A, except that the values of x_d and x_u are interchanged. For each case, Eq. [3] was used to generate values of T_d/T_u at 1-s intervals from $t = 1\text{ s}$ to $t = 99\text{ s}$. These values of T_d/T_u were then used in Eq. [5] to obtain estimates of the flux as a function of time for $t > 0$. Flux estimates were also obtained from Eq. [7] for $t > t_0/2$ by using the generated values of T_d/T_u , as well as the time t corresponding to each value of T_d/T_u . Flux estimates from Eq. [5] and [7] were then compared with the specified values of J (Table 1) used for generating the T_d/T_u time series.

RESULTS AND DISCUSSION

Figure 1 shows the values of T_d/T_u generated using Eq. [3] and the parameters in Table 1. For all three cases, T_d/T_u changes relatively rapidly with time at early times and then gradually approaches the asymptotic values predicted by Eq. [4]. The ratio

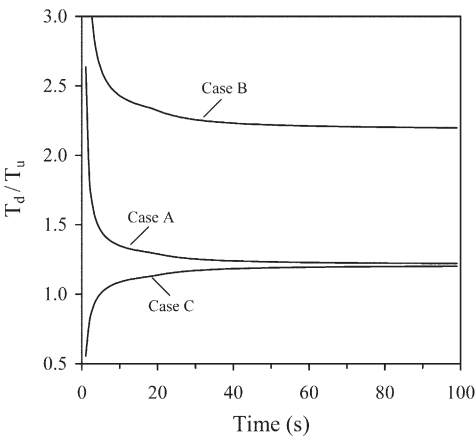


Fig. 1. Ratio of temperature increases at downstream and upstream positions (T_d/T_u) as a function of time for Cases A, B, and C. Results were obtained by using Eq. [3]. The parameters used for each case are given in Table 1.

x_d/x_u determines whether T_d/T_u increases or decreases with time. The spacing ratio is less than one for Cases A and B and greater than one for Case C. Additional calculations (results not shown) revealed that T_d/T_u becomes more nonlinear in time as x_d/x_u departs farther from unity and that T_d/T_u is constant (no time dependence) for the case of equidistant probe spacing (i.e., when $x_d = x_u$). The latter result is consistent with the observation of Wang et al. (2002) that Eq. [3] simplifies to $T_d/T_u = \exp(Vx/\kappa)$ for the case of equidistant probe spacing, with $x = x_d = x_u$. Unfortunately, the simple form that results when $x_d = x_u$ cannot be used in practice because small differences between the probe spacings inevitably arise during sensor construction.

The time dependence of T_d/T_u is reflected in the flux estimates obtained from both Eq. [5] and Eq. [7] (Fig. 2). Both expressions yield relatively poor approximations of the specified flux (dashed horizontal lines) at early times when T_d/T_u exhibits its greatest nonlinearity but yield better approximations at later times as T_d/T_u approaches its asymptotic value. For very early times (the first few seconds after time $t = t_0/2$), flux estimates from Eq. [5] are more accurate than those from Eq. [7] (results not shown in Fig. 2). For all times shown in Fig. 2, however, soil water flux is estimated more accurately with Eq. [7] than with Eq. [5]. Caution must be exercised when using Eq. [7] to estimate flux at relatively early times. To improve the accuracy of Eq. [7] at early times, it is necessary to add higher-order terms to more completely account for the finite duration of the heat pulse.

Ochsner et al. (2005) calculated their ratio-method flux estimates by using measurements of T_d/T_u for times in the interval 40 s < t < 50 s. At the midpoint of this interval (i.e., $t = 45$ s), fluxes estimated from Eq. [7] agree exceptionally well with the specified flux for all cases. Fluxes from Eq. [7] for $t = 45$ s are 4.71, 19.11, and 4.69 cm h⁻¹ for Cases A, B, and C, respectively. These estimates deviate from the specified fluxes (Table 1) by amounts of 0.0107, 0.0112, and -0.0107 cm h⁻¹, respectively (relative errors of 0.23%, 0.06%, and -0.23%, respectively). Fluxes from Eq. [5] for $t = 45$ s are 5.19, 19.59, and 4.21 cm h⁻¹ for Cases A, B, and C, respectively. These estimates deviate from the specified fluxes by amounts of 0.492, 0.493, and -0.492 cm h⁻¹, respectively (relative errors of 10.5%, 2.6%, and -10.5%, respectively). Equation [7] clearly yields more accurate estimates than Eq. [5] for this sampling time of $t = 45$ s. Furthermore, the accuracy achieved with Eq. [7] suggests that it is an excellent alternative to the exact model when used at this sampling time.

Although a complete analysis of the error associated with the use of Eq. [7] is beyond the scope of this manuscript, two general observations can be made. First, it is clear from the results presented in Fig. 2 that the error in flux estimates obtained using Eq. [7] is inversely proportional to time t at which T_d/T_u is measured. Earlier sampling times result in larger error, and later sampling times result in smaller error. Second, it is evident from the manner in which Eq. [7] was derived that the error in flux estimates obtained using Eq. [7] is directly proportional to t_0 , the length of the heating duration. Smaller heating durations result in smaller error, and larger heating durations result in larger error.

CONCLUSIONS

Wang et al. (2002) proposed an expression to estimate soil water flux from the temperature increase ratio (T_d/T_u) measured with the three-probe sensor of Ren et al. (2000). A distinct advantage of this expression (Eq. [5]) is its simple algebraic form. It approximates an exact model that is far more difficult to use in practice. We have derived an improved approximation (Eq. [7]) to estimate soil water flux from T_d/T_u . Because it accounts for the time dependence of T_d/T_u , Eq. [7] estimates flux with greater accuracy than Eq. [5]. The improved approximation also retains the advantage of simple algebraic form.

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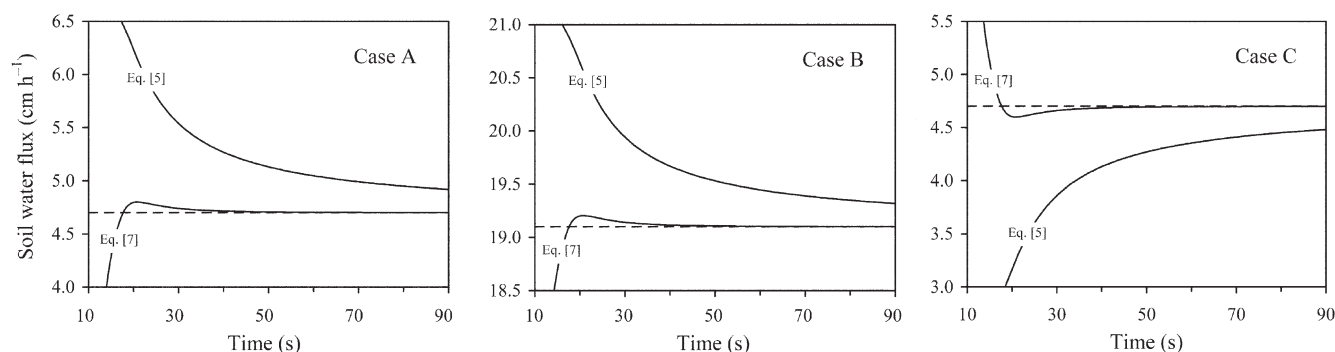


Fig. 2. Estimates of soil water flux (J) as a function of time for Cases A, B, and C. Results were obtained by using the values of T_d/T_u from Eq. [3] (see Fig. 1) in Eq. [5] and [7]. The parameters used for each case are given in Table 1. The horizontal dashed line in each panel represents the specified value of the flux.